selected for publication, but owing to the fact the December Number had to be cut short in order to get it out without further delay their solutions were omitted.—EDITOR.]

PROBLEMS.

25. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find, if possible, integral values of each of the seven linear measurements of a rectangular parallelopiped; i. e. length, breadth, height, the diagonals of each of the three different rectangular sides, and the diagonal from an upper corner to the opposite lower corner; or, find integral values, if possible, of a, b, c, d, e, f, and g, as shown in the equations,—$a^2+b^2=c^2$, $a^2+d^2=e^2$, $a^2+f^2=g^2$, $b^2+d^2=f^2$, $b^2+e^2=g^2$, $c^2+d^2=g^2$, $c^2+e^2=f^2$. If not possible, how many of them can have integral values? and which?

26. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find (1) a *square fraction* the arithmetical difference of whose terms is a *cube*; and (2) find a *cubic fraction* the arithmetical sum of whose terms is a *square*.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

12. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

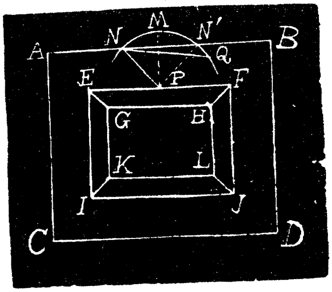
A large plane area is ruled by two sets of parallel equidistant straight lines, the one set perpendicular to the other. The distance between any two lines of the first set is a; the distance between any two lines of the second set is b. If a regular polygon of $2n$ sides be thrown at random upon this area, find the chance that it will fall across a line, the diameter of the circum-circle of the polygon being less than a or b.

II. Solution by H. W. DRAUGHON, Clinton, Louisiana.

In the rectangle $ABCD$ let $AB=a$ and $AC=b$. Let c=apothem of polygon, and r=radius of its circum-circle.

Let the sides of the rectangle $EFIJ$, be parallel to, and distant c, from the corresponding sides of $ABCD$, and let the sides of the similarly

placed rectangle $GHKL$, be r distant from the corresponding sides of AB. Draw EG, FH, LJ, and IK.



1. If the center of the polygon falls without the rectangle $EFIJ$, the polygon will cross a line in every possible position. $\therefore$ the number of favorable positions for this case is, $n_1 = 2\pi r \times$ area of surface on which center falls $= 2\pi r[ab-(a-2c)(b-2c)]$. 2. Let us suppose that the center falls on the point P within the trapezoid $EFGH$. From P as a center, radius r, draw an arc cutting AB in the points N and N'. Draw one side of polygon NQ, also draw PM, perpendicular to AB Put $AM=x$ and $PM=y$. The number of favorable positions for the point P, is obviously, $2n \times \operatorname{arc} NN' = 4nr\cos^{-1}\left(\frac{y}{r}\right)$.

When x varies from $a-r$ to r, y can have any value from r to c. When x varies from r to c, y can have any value from x to c. The integration between the remaining limits for x and y, will obviously give the same result as that between last mentioned limits.

$\therefore$ in this case, the total number of favorable positions is,

$$n_2 = 4nr\left[2\int_c^r\int_c^x \cos^{-1}\left(\frac{y}{r}\right)dxdy + \int_r^{a-r}\int_c^r \cos^{-1}\left(\frac{y}{r}\right)dxdy\right]$$

$$= 4nr\left[2\int_c^r (x\cos^{-1}\left(\frac{x}{r}\right)dx - \sqrt{(r^2-x^2)}dx - c\cos^{-1}\left(\frac{c}{r}\right)dx\right.$$

$$\left. + \sqrt{(r^2-c^2)}\,dx + \int_r^{a-r}(-c\cos^{-1}\left(\frac{c}{r}\right)dx + \sqrt{(r^2-c^2)}\,dx)\right]$$

$$= 4nr\left\{2\left[\left(\frac{x^2}{2}-\frac{r^2}{4}\right)\cos^{-1}\left(\frac{x}{r}\right) - r^2\cos^{-1}\left(\frac{x}{r}\right) + \frac{x}{2}\sqrt{(r^2-x^2)}\right.\right.$$

$$\left.\left. + \frac{r^2}{2}\cos^{-1}\left(\frac{x}{r}\right)\right]_c^r + \left[-cx\cos^{-1}\left(\frac{c}{r}\right) + x\sqrt{(r^2-c^2)}\right]_r^{a-r}$$

$$= 4nr[(\tfrac{3}{2}r^2 + 2cr - ac - c^2)\cos^{-1}\left(\frac{c}{r}\right) + (a-2r-c)\sqrt{(r^2-c^2)}].$$

If the center falls within rectangle $GHKL$, the polygon can not cross a line.

3. The total number of favorable positions, when the center falls within the trapezoid $GKEI$ is found by changing a to b in the value of n_2. $\therefore$ we have for this case, number of favorable positions,

$$n_3 = 4n[(\tfrac{3}{2}r^2 + 2cr - bc - c^2)\cos^{-1}\left(\frac{c}{r}\right) + (b-2r-c)\sqrt{(r^2-c^2)}].$$

4. If the center falls within the trapezoids $KLIJ$ or $HLFJ$, the number of favorable positions is, respectively, n_2 and n_3. $\therefore$ when the center falls within rectangle $ABCD$, the number of favorable positions is,

$n_4 = n_1 + 2n_2 + 2n_3 = 2\pi r[ab-(a-2r)(b-2r)]$

$$+8nr[(\tfrac{3}{2}r^2+2cr-ac-c^2)\cos^{-1}\left(\frac{c}{r}\right)+(a-2r-c)\sqrt{(r^2-c^2)}]$$

$$+8nr[(\tfrac{3}{2}r^2+2cr-bc-c^2)\cos^{-1}\left(\frac{c}{r}\right)+(b-2r-c)\sqrt{(r^2-c^2)}]$$

$$=2\pi r[ab-(a-2r)(b-2r)]+8nr[(3r^2+4cr-(a+b)c-2c^2)$$

$$\cos^{-1}\left(\frac{c}{r}\right)+(a+b-4r-2c)\sqrt{(r^2-c^2)}].$$

Let S be the number of rectangles, ab, then the probability required is, $P=Sn_4\div 2\pi rSab=n_4\div 2\pi rab=\{\pi[ab-(a-2r)(b-2r)+4n[(3r^2+4cr$

$$-(a+b)c-2c^2)\cos^{-1}\left(\frac{c}{r}\right)+(a+b-4r-2c)\sqrt{(r^2-c^2)}]\}\div\pi ab.$$

PROBLEMS.

24. Proposed by F. P. MATZ, M. Sc. Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

The average area of the triangle formed by three perpendiculars drawn from the sides of the triangle (a, b, c), is $\mathbf{A}=(a^4+b^4+c^4)/48\triangle$.

25. Proposed by G. B. M. ZERR, Principal of High School, Staunton, Virginia.

The probability that the distance of two points taken at random in a given convex area A shall exceed a given limit (a) is

$$\triangle=\frac{1}{3A^2}\int\int(C^3-3a^2C+2a^3)dpd\theta,$$

where C is a chord of the area, whose co-ordinates are p, θ; the integration extending to all values of p, θ, which give a chord $C>a$. What is $\triangle$ when the area is a circle? If in the circle $a=r=$radius $\triangle=\dfrac{3\sqrt{3}}{4\pi}$.

INFORMATION.

PROFESSOR ARTHUR CAYLEY DEAD.

The Distinguished English Mathematician Passes Away at Cambridge.

LONDON, Jan. 31.—Prof. Arthur Cayley, of the University of Cambridge, died to-day, in the seventy-fourth year of his age. He had been for thirty-two years Sadlerian professor of pure mathematics at Cambridge, and